

DICEY 21

MATHEMATICAL ANALYSIS OF "DICEY 21" PREPARED FOR LUCKY GAMES L.L.C.

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DICEY 21 MATHEMATICAL ANALYSIS

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Introduction and General Procedure:

1. "Dicey 21" and "Chuck-A-Luck PLUS" are two wagers played concurrently, using six standard 52-card decks of playing cards and three community dice, two of the same color and a third off-color.
2. The round begins with players placing any combination of the Dicey 21 main wager (hereafter referred to as the "21 Wager") and up to three of the six numbered spots of the Chuck-A-Luck PLUS Wager.
3. Each player is then dealt one card face-up, which is used for both the 21 Wager and the Chuck-A-Luck PLUS Wager.
4. The dealer then begins from his left, and each player who placed the 21 Wager will then play his hand in the manner indicated in the section entitled "21 Wager Rules".
5. Once all players who have played the 21 Wager have completed their hand, the dealer will roll the community dice, and resolve the 21 Wager and Chuck-A-Luck PLUS wager, using the two similar color dice for the 21 Wager and all three dice, along with the first (or only) card in a player's hand for the Chuck-A-Luck PLUS wager, in the manner indicated in the section entitled "Chuck-A-Luck PLUS Wager Rules".
6. After the resolution of all wagers, the dealer collects the cards and a new round of wagering resumes.

21 Wager Rules:

1. After each player has been given their first card, they are given the opportunity to make a Raise Wager, up to double the amount of their 21 Wager, before they begin playing their hand.
2. After choosing whether or not to raise, players may stand, or draw additional cards until they are either satisfied with their hand, or reach a total of Hard 19 or more.
3. Once all players have finished drawing, the dealer rolls the community dice. Each player's hand total will consist of the totals of the two similar color dice and any cards in their hand, calculated in the standard blackjack fashion, and compared to the following payable:

- a. If the player's total is 18, 19 or 20, the 21 Wager and Raise Wager will pay even money
- b. If the player's total is 21,
 - i. If the player stood pat on their original one-card hand, the player will be considered to have a natural, and their 21 Wager and Raise Wager will be paid at a rate of 2:1.
 - ii. If the player has multiple cards, their 21 Wager and Raise Wager will be paid at a rate of 6:5.
- c. If the player's total is 17, the 21 Wager and Raise Wager will push.
- d. Otherwise, the player's 21 Wager and Raise Wager will lose.

Chuck-A-Luck PLUS Wager Rules:

1. Once the community dice are rolled, the three dice and the player's first card (or only card, if the player only has one) are consulted, and each number wagered on is checked for a winner in the following fashion:
 - a. If all three dice match a number the player has wagered on, and the player's first (or only) card matches the same rank as the selected number, that player scores a Quad.
 - b. If all three dice match a number the player has wagered on, and the player's first (or only) card is a King or Queen, that player scores a Royal.
 - c. If all three dice match a number the player has wagered on, yet the player's first (or only) card is neither a King, Queen or of the same rank, that player scores a Triple.
 - d. If any two dice match a number the player has wagered on, that player scores a Double.
 - e. If one dice matches a number the player has wagered on, that player scores a Single.
2. If none of the dice match a number the player has wagered on, the player loses that wager.

Methodologies:

1. A spreadsheet with an infinite deck model was created, analyzing all possible outcomes to determine the player's expectation on the 21 Wager and the Chuck-A-Luck PLUS Wager.
2. A computer program was created in the C# programming language (VS 2008 standard) to simulate 1,000,000,000 hands of play under standard casino play conditions (a six deck shoe, dealt four and a half decks deep) in order to verify the results above, as well as collate various miscellaneous statistics.
3. In all cases, the player is assumed to be playing with the optimal strategy on all bets wagered.

Optimal Player Strategy:

1. The player should make the maximum (double) raise with a first card of an Ace, Two, Three, or any ten value.
2. For hard hands, hit on any total between two and eight, and stand on any other hard total.
3. For soft hands, hit on any total between 15 and 18, and stand on any other soft total.

Results:

1. On the 21 Wager, with optimal player strategy, the house will win, on average, ***an amount equal to 2.41% of the 21 Wager, or 1.17% of the total of the 21 and Raise Wagers.***
2. The infinite deck model calculated an average house win of ***2.39% of the 21 wager, or 1.15% of the total of the 21 and Raise Wagers.*** However, as the simulation was conducted under practical conditions, the value of the simulation will be taken as correct, with the infinite deck model serving to verify the results of the simulation.
3. On the Chuck-A-Luck PLUS Wager, the house will win, on average, ***an amount equal to 3.21% of the Chuck-A-Luck PLUS Wager.***

Report completed on 10 February, 2010 by Charles Mousseau, B.Sc.



Results are deemed reliable.

DICEY 21: APPENDIX OF TABLES

(Note that in the tables below, the final result is expressed as the return for the player. As this is a negative number, this indicates a house advantage, equal to the equivalent positive amount. For instance, a player return of -3.21% corresponds to a 3.21% house advantage.)

Table #1: Calculation detail for Chuck-A-Luck PLUS Wager.

"Chuck-a-Luck PLUS" bet					
<u>Result</u>	<u>Occurs</u>	<u>P(Occur)</u>	<u>Payout</u>	<u>Value</u>	<u>Volatility</u>
No Match	125	0.578703704	-1	-0.578704	0.102714
Single	75	0.347222222	1	0.347222	0.147958
Double	15	0.069444444	2	0.138889	0.240537
Triple(*)	1	0.00462963	13.1	0.060541	0.784381
SUM	216	1		-3.21%	1.28
				Return	1.13
					Variance
					Std Dev

(*) Calculations for value of "Triple Matches"

.. When player's card is:

<u>Result</u>	<u>Occurs</u>	<u>P(Occur)</u>	<u>Payout</u>	<u>Value</u>
Royal	48	0.153846154	20	3.076923
Quad	24	0.076923077	40	3.076923
Other	240	0.769230769	9	6.923077
SUM	312	1		13.076923

Table #2a: Simulation result of Ante and 21 Wagers

<u>First Card:</u>	<u>EV</u>	<u>Bet Size</u>	<u>Value</u>
X	-0.00001091	3	-0.00003274
A	0.017077549	3	0.051232647
2	0.001224268	3	0.003672803
3	0.001432645	3	0.004297936
4	-0.00088777	1	-0.00088777
5	-0.00599908	1	-0.00599908
6	-0.01218152	1	-0.01218152
7	-0.0186293	1	-0.0186293
8	-0.02422105	1	-0.02422105
9	-0.02132174	1	-0.02132174
		Total:	-2.41%
		Final bet:	2.052418845
		Return:	-1.17%

Table #2b: Calculated result from infinite deck model of Ante and 21 Wagers

First Card	EV	Bet Size	Value
X:	0	3	0
A:	0.017094017	3	0.051282051
2:	0.001230734	3	0.003692202
3:	0.001431847	3	0.00429554
4:	-0.00090506	1	-0.000905062
5:	-0.00597061	1	-0.005970607
6:	-0.01207649	1	-0.012076491
7:	-0.01856572	1	-0.018565721
8:	-0.02426036	1	-0.024260355
9:	-0.02136752	1	-0.021367521
		Total:	-2.39%
		Final Bet:	2.076923077
		Return:	-1.15%

Table #3a: 21 Wager, simulation result, win/loss/tie breakdown.

Result (EV)	P(Result)	p * EV	Volatility
-3	0.2341907	-0.702572	2.1077160
-1	0.2590548	-0.2590548	0.2590548
0	0.1137272	0	0.0000000
1	0.1239537	0.1239537	0.1239537
1.2	0.0396571	0.0475885	0.0571062
2	0.0021361	0.0042721	0.0085443
3	0.1880044	0.5640133	1.6920399
3.6	0.0158029	0.0568904	0.2048053
6	0.0234732	0.140839	0.8450338
Totals:	1	-2.41%	5.30
Standard Deviation:			2.30

Table #3b: 21 Wager, miscellaneous statistics.

1 total cards:	0.460908638
2 total cards:	0.426741218
3 total cards:	0.100684357
4 total cards:	0.011034794
5 total cards:	0.00061453
6 total cards:	0.000016298
7 total cards:	0.000000165
Average:	2.17 cards
Loss:	49.32%
Push:	11.37%
Win	39.30%

